# Nonlinearity Estimation and Spectral Regrowth Prediction of Power Amplifiers using Correlation Techniques

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*Abstract* Nonlinearity estimation and spectral regrowth prediction are crucial to power amplifier linearization. The correlation techniques shown here can estimate amplifier nonlinearity and predict out-of-band power spectrum. Crosscorrelation is performed between the output envelope and a test sequence generated from the input signal and can be carried out in background during amplifier operations. The proposed method is applicable to multi-channel DS-CDMA systems and IS-95 forward-link CDMA signals are employed as an example.

*Index Terms* — spectral analysis, power amplifiers, nonlinearity, amplifier characterization, correlation.

#### I. INTRODUCTION

Power amplifier nonlinearity can lead to loss of signal accuracy (as characterized by error-vector magnitude) and to generation of interference in adjacent frequency channels as characterized by the adjacent channel power ratio (ACPR). Traditionally, single tone or two-tone testing is used to extract the amplifier nonlinearity from AM-AM and AM-PM distortion or intermodulation distortion measurements [1-3]. Employing these measurements, the spectral regrowth of complex digital modulation signals can be predicted by various behavioral models [4-5]. However, these approaches require special input signal generation, and are generally not applicable for characterization of amplifiers during real-time operation. An alternate method is to capture the input and output time domain waveforms to extract the nonlinearity and use FFTs to calculate the output spectrum under operating conditions [6]. This approach is intensive in computation complexity and hardware requirements. In this paper, we demonstrate simple correlation techniques that can estimate power amplifier nonlinearity and predict the out-of-band spectral regrowth. The correlation method can be carried out without interrupting the amplifier's normal operation and can potentially be carried out with simple analog circuitry.

In what follows, the correlation measurement technique is first reviewed. The calculation of spectral regrowth based on amplifier nonlinearity coefficients is then described. Finally, the correlation technique for spectral regrowth estimation is demonstrated by simulation and measurement.

## II. CORRELATION METHODOLOGY FOR NONLINEARITY ESTIMATION

The basic correlation methodology is illustrated in Fig.1 [7]. In a multi-channel DS-CDMA system, the PA's baseband input signal is the sum of several uncorrelated PN (pseudorandom noise) digital binary sequences  $S_i$  (i is the channel number). A new binary test signal  $S_{test}$  is created by forming the product of  $S_i$  for three inputs. Both  $S_{test}$  and the output signal  $V_{out}$  are nonlinear functions of the input signal. We have shown [7] that the nonlinearity of the power amplifier can be extracted by the maximal correlation values between  $V_{out}$  and  $S_{test}$ :

$$\operatorname{Max}(\Phi_{Vout,Stest}(\tau)) = \operatorname{Max}(\lim_{T \to \infty} \frac{1}{T} \int_{0}^{\tau} V_{out}(t) S_{test}(t+\tau) dt) \propto \operatorname{nonlinearity}(1)$$

The test signal can be inherently generated from the actual baseband CDMA signals such as IS-95 forward link signals.



Fig.1 Correlation methodology

## III. SPECTRAL REGROWTH AND CORRELATION ANALYSIS WITH POLYNOMIAL NONLINEARITY

#### A. Nonlinearity model

The quasi-memoryless nonlinearity of a PA can be characterized in various ways, including a) a complex polynomial model [4] or b) the Saleh model [8]. Both approaches are considered here. We used CW single tone measurements for a commercial wireless LAN amplifier, Intersil's ISL3990 GaAs PA, to obtain the model fitting parameters. Coefficients for a five-order complex

polynomial were obtained by curve fitting as shown in Fig.2, in accordance with,

$$\widetilde{V}_{out} = \widetilde{V}_{in} (\widetilde{a}_1 + \widetilde{a}_3 |V_{in}|^2 + \widetilde{a}_5 |V_{in}|^5) \cdot$$
(2)

The Saleh model in its polar format has four nonlinearitycontrol parameters, as shown in equation (3). Note that if the input amplitude r is very small or very large, the amplitude function A(r) is proportional to r or 1/r and phase function  $\Phi(r)$  approaches zero or a constant. The measured data and Saleh model fitting curves are shown in Fig.3.

$$A(r) = \alpha_a r / (1 + \beta_a r^2)$$

$$\Phi(r) = \alpha_{\phi} r^2 / (1 + \beta_{\phi} r^2)$$
(3)



Fig.2 AM-AM and AM-PM curves of measured (solid blue) and 5<sup>th</sup> order polynomial fitting (dashed red)



Fig.3 AM-AM and AM-PM curves of measured (solid blue) and Saleh fitting (dashed red)

#### B. Analytical spectral regrowth formulation

Spectral regrowth can be estimated from amplifier nonlinearity, together with input signal characteristics. We consider the input and output signals x(t) and y(t) in their rectangular format with polynomial nonlinear relationships as shown in Fig.4. The output autocorrelation  $\widetilde{R}_{\overline{y}\overline{y}}$  can be written as a function of the input autocorrelation  $\widetilde{R}_{\overline{y}\overline{y}}$  with different orders as:

$$\widetilde{\mathsf{R}}_{\widetilde{y}\widetilde{y}}(\tau) = P_1 \widetilde{\mathsf{R}}_{\widetilde{x}\widetilde{x}}(\tau) + P_3 \widetilde{\mathsf{R}}^*_{\widetilde{x}\widetilde{x}}(\tau) \widetilde{\mathsf{R}}^2_{\widetilde{x}\widetilde{x}}(\tau) + P_5 \widetilde{\mathsf{R}}^{*2}_{\widetilde{x}\widetilde{x}}(\tau) \widetilde{\mathsf{R}}^3_{\widetilde{x}\widetilde{x}}(\tau)$$
(4)

where  $\tilde{R}^*_{\bar{x}\bar{x}}$  is the complex conjugate of  $\tilde{R}_{\bar{x}\bar{x}}$  and  $P_1$ ,  $P_3$  and  $P_5$  represent the coefficients of different order of input autocorrelation.  $P_3$  and  $P_5$  are functions of the complex polynomial coefficients in (2) as:

 $P_3 = K_1(|\widetilde{a}_3|^2 + K_2|\widetilde{a}_5|^2 + 2K_2\Re\{\widetilde{a}_3\widetilde{a}_5^*\}), P_5 = K_3|\widetilde{a}_5|^2 \quad (5)$ where  $\Re$  denotes the real part and  $K_1$ ,  $K_2$  and  $K_3$  are constant numbers.

$$\widetilde{V}_{in} = I + j * Q = x(t)$$

$$NL$$

$$\widetilde{V}_{out} = \widetilde{V}_{out\_I} + j * \widetilde{V}_{out\_Q}$$

$$= y(t)$$

$$= y(t)$$

Fig.4 Complex polynomial input-output nonlinearity

Equation (4) is derived following the methods of references [4] and [9]. Simulation results in Fig.5 verify its correctness: The input signal x(t) is five-channel filtered PN sequences with Gaussian-like distribution and its autocorrelation is calculated. The PA is characterized by the coefficients obtained from curve fitting in Fig.2. The output autocorrelation  $\tilde{R}_{yy}$  is obtained by two approaches. The curve with blue circles is directly calculated from the sampled output signal y(t), while the curve with red stars is obtained by (4). Agreement of the two is excellent.



Fig.5 Output autocorrelation values of direct calculation (blue circles) and equation calculation (red stars)

The output power spectrum is the Fourier transform of output autocorrelation  $\tilde{R}_{\bar{y}\bar{y}}$ . Fig.6 shows the normalized Fourier transforms of the 3<sup>rd</sup> and 5<sup>th</sup> order input autocorrelation in (4). It can be seen that the adjacent and alternate channel leakage power is due to the 3<sup>rd</sup> and 5<sup>th</sup> nonlinearity, and thus is largely proportional to  $P_3$  and  $P_5$  respectively.



Fig.6 Fourier transforms of 3<sup>rd</sup> order input autocorrelation (solid blue) and 5<sup>th</sup> order input autocorrelation (dashed red)

## C. Analytical formulation of correlation functions

The in-phase and quadrature nonlinearity of the polynomial nonlinear block in Fig.4 can be extracted by proper correlation between the test signal  $S_{test}$  and the amplifier output signals [7]. The in-phase and quadrature correlations are defined as

$$\begin{cases} I\_correlation = \phi_{\text{Vout\_I, Stest\_I}}(\tau) \\ Q\_correlation = \phi_{\text{Vout\_I, Stest\_Q}}(\tau) \end{cases}$$
(6)

The maximal power correlation values are defined as

 $P_{corr} = Max(I\_correlation)^2 + Max(Q\_correlation)^2$ . (7) The derived maximal 3<sup>rd</sup> and 5<sup>th</sup> order power correlation values can be written in a manner very similar to (5) (for the spectral regrowth) as:

$$P_{3} = K_{1}'(|\tilde{a}_{3}|^{2} + K_{2}'|\tilde{a}_{5}|^{2} + 2K_{2}'\Re\{\tilde{a}_{3}\tilde{a}_{5}^{*}\}), P_{5} = K_{3}'|\tilde{a}_{5}|^{2}$$
(8)

where  $K'_1, K'_2$  and  $K'_3$  are constant numbers. Equations (5) and (8) show that both the out-of-band power and the maximal power correlation values have very similar dependence on the polynomial coefficients. Thus it is plausible that the correlation values are excellent indicators of the system nonlinearity and spectral regrowth.

## IV. SIMULATION AND MESUREMENT RESULTS

Matlab simulation was used to verify the similarity of spectral regrowth and power correlation values. The setup was similar to Fig.1 with five filtered PN sequences passing through a behavioral PA model, which can be a complex polynomial or Saleh model. The nonlinearity characteristics of PA can be changed by adjusting the parameters of its nonlinear models. Fig.7 and Fig.8 show the calculated three-dimension contours of the ACPR and the power correlation values vs. coefficients  $\tilde{a}_3$  in (2) or parameters  $\beta_a$  and  $\alpha_{\phi}$  in (3).



Fig.7 (a) ACPR@885kHz (dBc) and (b)  $3^{rd}$  power correlation values (dB) vs.  $\tilde{a}_{3}$ 



Fig.8 (a) ACPR@885kHz (dBc) and (b)  $3^{rd}$  power correlation values (dB) vs.  $\beta_a$  and  $\alpha_{\phi}$ 

The above results show that the correlation extraction method can predict the spectral regrowth phenomena quite well. The experimental test environment is shown in Fig.9. The baseband input signals are generated with Matlab and downloaded to an Agilent ESG waveform generator as the RF input of the PA under test. The baseband output signals are collected by an Agilent PSA spectrum analyzer and VSA vector signal analyzer. The PA's equivalent nonlinearity characteristics can be changed by using a baseband polynomial predistorter (PD).



Fig.9 Measurement setup

The correlation measurements allow adjusting  $\tilde{b}_3$  coefficients to minimizing ACPR. The three-dimension contours of ACPR and power correlation values vs. the predistorter's coefficients  $\tilde{b}_3$  are shown in Fig.10. The similarity of these characteristics confirms the preceding simulations. Moreover, they demonstrate that the ACPR can be minimized by experimental minimization of the correlation values.



Fig.10 (a) ACPR@750kHz (dBc) and (b)  $3^{rd}$  power correlation values (dB) vs.  $\tilde{b}$ ,

### IV. SUMMARY

Simple correlation techniques have been used to estimate power amplifier nonlinearity and predict its spectral regrowth. The potential of this method was shown both by simulation and by experiment. The technique is potentially applicable to the measurement of amplifier nonlinearity in real time during amplifier operation.

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#### REFERENCES

- [1] C. J. Clark, C. P. Silva, A. A. Moulthrop and M. S. Muhal, "Power-amplifier Characterization Using a Two-tone Measurement Technique," *IEEE Trans. MTT.*, Vol 50, no. 6, pp. 1590-1602, Jun. 2002.
- [2] S.W. Chen, W. Panton and R. Gilmore, "Effects of Nonlinear Distortion on CDMA Communications Systems," *IEEE Trans. MTT.*, Vol. 44, no. 12, pp. 2743-2750, Dec. 1996.
- [3] H. Ku, M.D. McKinle and J. S. Kenney, "Extraction of Accurate Behavioral Models for Power Amplifiers with Memory Effects using Two Tone Measurements," 2002 *IEEE MTT-S IMS Digest*, pp. 139-142, Jun. 2002.
- [4] K. Gard, M. B. Steer and L. E. Larson, "Generalized Autocorrelations of Spectral Regrowth from Bandpass Nonliear Circuits," 2001 *IEEE MTT-S IMS Digest*, pp. 9-12, Jun. 2001.
- [5] G. T. Zhou and J. S. Kenney, "Predicting Spectral Regrowth of Nonlinear Power Amplifiers," *IEEE Trans. Communications.*, Vol 50, no.5, pp. 718-722, May. 2002.
- [6] E. G. Jeckeln, F.M. Ghannouchi, and M.A. Sawan, "A New adaptive Predsitortion Techniques Using Software-Defined Radio and DSP Technologies Suitable for Base Station 3G Power Amplifiers," IEEE Trans. MTT., Vol 9, no. 9, pp. 2139-2148, Jun. 2002.
- [7] M. Y. Li, I. Galton, L.E. Larson, P.M. Asbeck, "Correlation Techniques for Estimation of Amplifier Nonlinearity," 2004 IEEE RAWCON Digest, pp. 179-182, Sep. 2004.
- [8] A. M. Saleh, "Frequency-Independent and Frequencu-Dependent Nonlinear Models of TWT Amplifiers," IEEE Trans. Communications, Vol 29, no.11, pp. 1715-1720, Nov. 1981.
- [9] S.P. Stapleton, G.S. Kandola, and J.K. Cavers, "Simulation and Analysis of an Adaptive Predistorter Utilizing a Complex Spectral Convolution," *IEEE Trans. MTT.*, Vol. 41, no. 4, pp. 387-393, Nov. 1992.